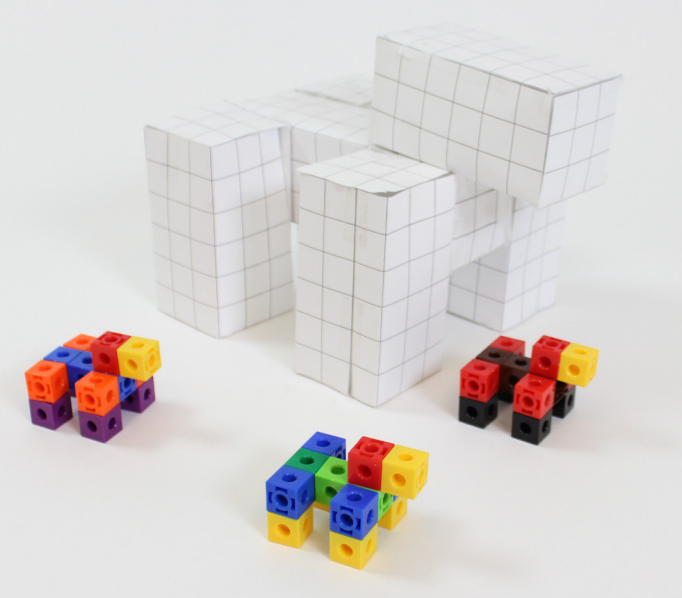

Unit Dog

LEVEL
Grade 6 or Grade 7 in a unit on surface area and volume

MODULE/CONNECTION TO AP*
Areas and Volumes

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MODALITY
NMSI emphasizes using multiple representations to connect various approaches to a situation in order to increase student understanding. The lesson provides multiple strategies and models for using those representations indicated by the darkened points of the star to introduce, explore, and reinforce mathematical concepts and to enhance conceptual understanding.

P – Physical
V – Verbal
A – Analytical
N – Numerical
G – Graphical

ABOUT THIS LESSON
In this lesson, students use linking cubes and nets to explore how the surface area and volume of an irregularly shaped object, "unit dog," change as the dimensions are scaled. The linking cubes are used to construct the "dog" and determine the surface area and volume by counting. Students create nets for the legs, head, and torso of the "dog." Scale factors are applied to the "unit dog" and students construct the larger dogs using nets and cardstock models. Students use ratios to determine how the surface area and volume are affected by the scale factor and create an expression that can be applied to any scale factor to calculate the surface area and volume for this irregular shape. This activity provides an engaging setting for students to practice and apply their skills with nets, surface area, volume, and scale factors.

This lesson can be used as a follow up activity to "Surface Area and Volume" where students explore the effects of scale factors on the surface area and volume of a cube using ratios.

OBJECTIVES
Students will

- determine the surface area and volume of an irregularly shaped object.
- draw front, side, and top views of the object.
- create nets for various parts of the object.
- apply scale factors to the nets.
- build models with unit cubes and with cardstock nets.
- investigate the resulting effects on surface area and volume when dimensions of a shape change proportionally.

TEACHER PAGES

COMMON CORE STATE STANDARDS FOR MATHEMATICAL CONTENT

This lesson addresses the following Common Core State Standards for Mathematical Content. The lesson requires that students recall and apply each of these standards rather than providing the initial introduction to the specific skill.

7.G.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
See questions 2-3, 8-9

6.G.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
See questions 5a-b, 6-7

Reinforced/Applied Standards

6.RP.3d: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
(d) Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
See questions 7-9

6.EE.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
See question 8's last row

COMMON CORE STATE STANDARDS FOR MATHEMATICAL PRACTICE

These standards describe a variety of instructional practices based on processes and proficiencies that are critical for mathematics instruction. NMSI incorporates these important processes and proficiencies to help students develop knowledge and understanding and to assist them in making important connections across grade levels. This lesson allows teachers to address the following Common Core State Standards for Mathematical Practice.

MP.3: Construct viable arguments and critique the reasoning of others.
Students compare a variety of student generated nets and then discuss how different nets can create the same 3-dimensional figure.

MP.5: Use appropriate tools strategically.
Students investigate using linking cubes, nets, and a table.

MP.8: Look for and express regularity in repeated reasoning.
Students use repeated calculations to develop general rules for determining surface area and volume using scale factors.

FOUNDATIONAL SKILLS

The following skills lay the foundation for concepts included in this lesson:

- Calculate surface area and volume of cubes and rectangular prisms
- Sketch nets of cubes and rectangular prisms

ASSESSMENTS

The following types of formative assessments are embedded in this lesson:

- Student cooperative learning groups create appropriately scaled models.
- Students engage in independent practice.
- Students summarize a process or procedure.

The following additional assessments are located on our website:

- Areas and Volumes – 6th Grade Free Response Questions
- Areas and Volumes – 6th Grade Multiple Choice Questions
- Areas and Volumes – 7th Grade Free Response Questions
- Areas and Volumes – 7th Grade Multiple Choice Questions

The following additional assessments are located on our website.

- Assessments by Course – 6th Grade
- Assessments by Course – 7th Grade

MATERIALS AND RESOURCES

- Student Activity pages
- 13 linking cubes for each student
- Copies of 1 inch square grid paper printed on cardstock
- Tape
- Scissors
- Sample unit dog made from linking cubes
- Sample unit dog made from grid paper
- Large copy of the chart provided in question 10
- Interactive applet that shows how the surface area and volume of a cube change as the lengths of the sides change: http://www.mathopenref.com/cubearea.html and http://www.mathopenref.com/cubevolume.html
- Interactive applet that shows front, back, side, and three-dimensional views of a rectangular prims and shows how the surface area and volume of a rectangular prism change as the lengths of the sides change: http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/
- Interactive applet that shows nets for a cube: http://mathworld.wolfram.com/Cube.html and http://www.mathsnet.net/geometry/solid/nets.html
- Interactive applet that allows the virtual construction of solids with unit cubes that can be rotated to examine front, back, side, and three-dimensional views: http://nlvm.usu.edu/en/nav/frames_asid_195_g_1_t_3.html?open=activities&from=topic_t_3.html

TEACHING SUGGESTIONS

The Unit Dog lesson should be teacher-led. Organize the students into small groups and lead the students through the following steps.

Question 1:
Demonstrate how to make a unit dog from linking cubes and then have each student build their own dog, using their 13 linking cubes. Some students may require a step-by-step process along with the demonstration to successfully build the "dog."

This part of the lesson provides an opportunity to review the vocabulary associated with a cube such as edge and face along with a review of how to calculate surface area and volume. Note: Larger dogs should not be built with linking cubes. They are heavy and very unstable.

Question 2:
Have students determine the volume of their unit dogs.

Question 3:
Generally students do not correctly determine surface area if they try to count. Give them time to think about a good procedure. Once students begin to have the correct answer, have them share their process with the rest of the class.

Question 4:
Have each student draw front, side, and top views of the unit dog.

Question 5:
Show the students how to draw the nets for the leg/head and torso. Use the linking cubes as a model to help students make the transition from a 3-dimensional object to a 2-dimensional net. Students may need to draw a net for a cube before creating the more complex nets for the "dog." Discuss the difference between the unit cube and the leg/head and torso to help them see a pattern and make the connection between the two shapes. Note: The lesson "Nets for Cubes" on the NSMI website provides pictures of 11 different ways to construct a net for a cube.

Question 6:
Students create different nets and compare these to ones drawn by classmates. Some students may need to cut out and fold the net to confirm whether or not the net is correct.

Question 7:
Have students sketch the nets using a scale factor of two. Use the linking cubes to help students visualize how the scale factor affects the leg/head and torso. To help students make this connection, explain that each square of the net in question 5 is 2 by 2 instead of 1 by 1. Working with the individual squares of the net help students understand how the net changes based on the scale factor.

Question 8 – 9:
To ensure the scaled nets will fit on the grid provide, display a net for all students to use. The nets provided in the answers for questions 5a and 5b will fit on the provide grids.

Question 10:
Before class begins, construct a scaled unit dog to use as a model. In order to create a variety of different-sized dogs, assign groups to build particular dogs from grid paper nets. Scaled net drawings from questions 7 – 9 should be used to construct the paper scaled nets. Help the students realize they are to construct one net for the dog's torso and that the other 5 nets (head and legs) are the same net. Each of the different body parts can be assigned to different group members. Note: Some students might construct their "dogs" but cutting separate pieces for each part of the body instead of building a net. Providing a sample of a net of the leg/head for a larger "dog" will help students from making this mistake.

Each group should calculate surface area and volume for their particular dog. If there are two groups for each size dog, then the groups can verify the answers of the other group. Once all of the dogs are built,

have students post their measurements on the large copy of the chart from question 10. As the students fill in the factors by which surface area and volume have increased, ask them how they determined the factors and how they used the factors in the calculations. Students can use ratios to calculate these factors. As a check for understanding, have the students predict the surface areas and volumes for dogs with dimensions six, seven, and ten times those of the unit dog before moving to the last row of the table. Showing the process in the third and last column of the table will help students see the patterns. Display all of the dogs.

Question 11:
Students use the expressions from the last row of the table to calculate the surface area and volume with very large scale factors.

You may wish to support this activity with TI-Nspire™ technology. See *Working with Fractions and Decimals and Calculations Using Special Keys* in the NMSI TI-Nspire Skill Builders.

Suggested modifications for additional scaffolding include the following:

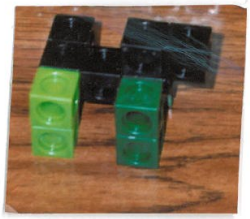
4 Provide a sketch of one of the requested views.
5 Provide a sketch of the net in part (a).
7 Provide a sketch of one of the required nets.

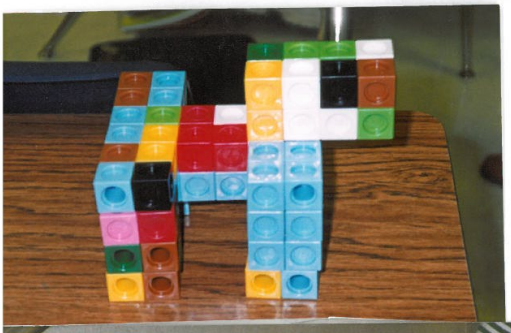
NMSI CONTENT PROGRESSION CHART

In the spirit of NMSI's goal to connect mathematics across grade levels, a Content Progression Chart for each module demonstrates how specific skills build and develop from third grade through pre-calculus in an accelerated program that enables students to take college-level courses in high school, using a faster pace to compress content. In this sequence, Grades 6, 7, 8, and Algebra 1 are compacted into three courses. Grade 6 includes all of the Grade 6 content and some of the content from Grade 7, Grade 7 contains the remainder of the Grade 7 content and some of the content from Grade 8, and Algebra 1 includes the remainder of the content from Grade 8 and all of the Algebra 1 content.

The complete Content Progression Chart for this module is provided on our website and at the beginning of the training manual. This portion of the chart illustrates how the skills included in this particular lesson develop as students advance through this accelerated course sequence.

3rd Grade Skills/ Objectives	4th Grade Skills/ Objectives	5th Grade Skills/ Objectives	6th Grade Skills/ Objectives	7th Grade Skills/ Objectives	Algebra 1 Skills/ Objectives	Geometry Skills/ Objectives	Algebra 2 Skills/ Objectives	Pre-Calculus Skills/Objectives	AP Calculus Skills/ Objectives
In a problem-solving situation with a real-world application, determine the area of rectangles and rectilinear figures with whole number side lengths by counting unit squares or multiplying side lengths.	In a problem-solving situation with a real-world application, determine the area of rectangles and rectilinear figures where one side is a fraction and one side is a whole number or a fraction. (Computations involving addition and subtraction of fractions are limited to like denominators.)	In a problem-solving situation with a real-world application, determine the area of rectangles and rectilinear figures with fractional and decimal side lengths. (These situations can include calculating area of rectangles on the coordinate plane.)	In a problem-solving situation with a real-world application, determine the area of rectangles, triangles, and composite figures with fractional and decimal side lengths. (These situations can include calculating area of rectangles and triangles on the coordinate plane.)	Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal line and one side on a vertical line, calculate the area of the figure.	Calculate the area of a triangle, rectangle, trapezoid, or composite of these three figures formed by linear equations and/ or determine the equations of the lines that bound the figure.	Calculate the area of a triangle, rectangle, trapezoid, circle, or composite of these figures formed by linear equations or equations of circles and/or determine the equations of the lines and circles that bound the figure.	Calculate the area of a triangle, rectangle, trapezoid, circle, or composite of these figures formed by linear equations, linear inequalities, or conic equations and/or determine the equations of the lines and circles that bound the figure.	Calculate the area of a triangle, rectangle, trapezoid, circle, or composite of these figures formed by linear equations, linear inequalities, or conic equations and/or determine the equations of the lines and circles that bound the figure.	Calculate the area between curves.
		In a problem-solving situation with a real-world application, determine the volume of rectangular prisms and non-overlapping right rectangular prisms with whole number edge lengths.	In a problem-solving situation with a real-world application, determine the volume of rectangular prisms and non-overlapping right rectangular prisms with fractional and decimal edge lengths and use scale factors to determine the volume of similar solids.	Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal or vertical line, calculate the surface area and/or volume of the cone or cylinder formed by revolving the bounded region about either of the lines.	Given the equations of lines (at least one of which is horizontal or vertical) that bound a triangular, rectangular, or trapezoidal region, calculate the surface area and/or volume of the solid formed by revolving the region about the line that is horizontal or vertical.	Given the equations of lines or circles that bound a triangular, rectangular, trapezoidal, or circular region, calculate the volume and/or surface area of the solid formed by revolving the region about a horizontal or vertical line.	Given the equations of lines or circles or a set of inequalities that bound a triangular, rectangular, trapezoidal, or circular region, calculate the volume and/or surface area of the solid formed by revolving the region about a horizontal or vertical line.	Given the equations of lines or circles or a set of inequalities that bound a triangular, rectangular, trapezoidal, or circular region, calculate the volume and/or surface area of the solid formed by revolving the region about a horizontal or vertical line.	Calculate the volume of a solid formed by revolving the region between two curves about a horizontal or vertical line.





TEACHER PAGES

Unit Dog

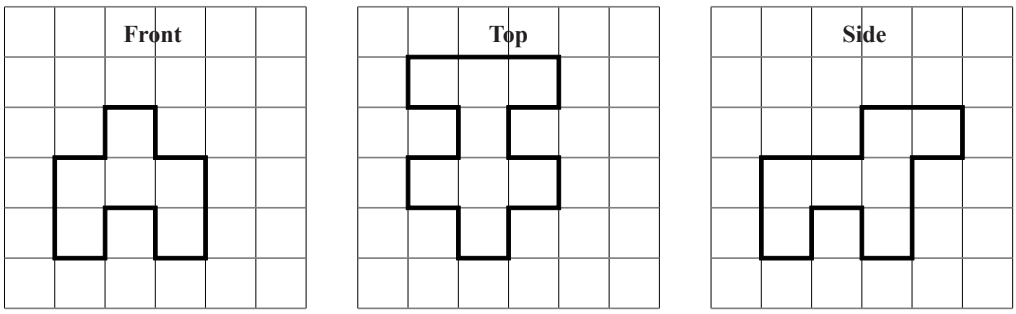
ANSWERS

1. The dog is constructed using 13 cubes.

2. The volume is 13 cubic units and can be determined by counting the number of cubes, each with a volume of 1 cubic unit.

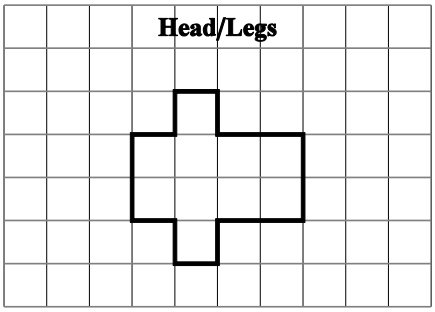
3. The surface area is 54 square units. The head and the four legs each have 9 exposed squares. The body has 9 exposed squares. The total surface area is 9 times 6 or 54 square units.

4.

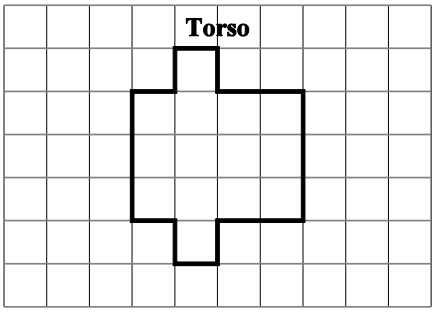


5. Many different nets are possible; one possible configuration is shown for the head/legs and torso

a.

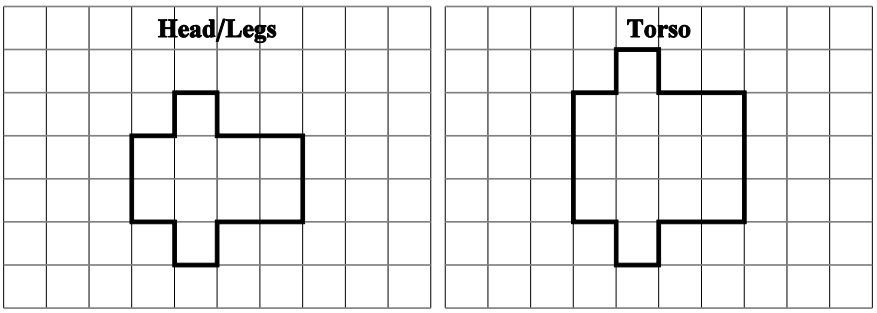


b.

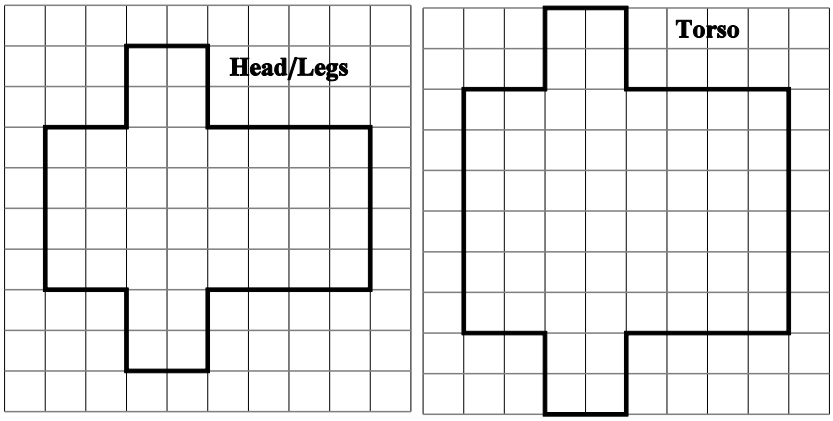


TEACHER PAGES

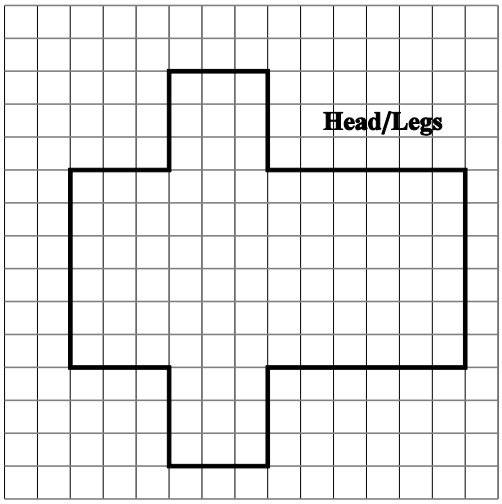
6.

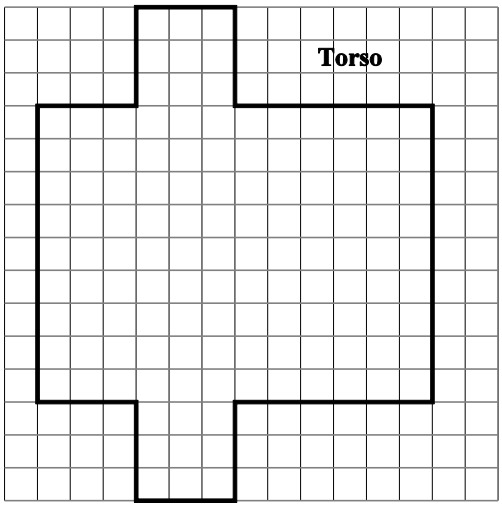


7. Sample nets for doubling each dimension.

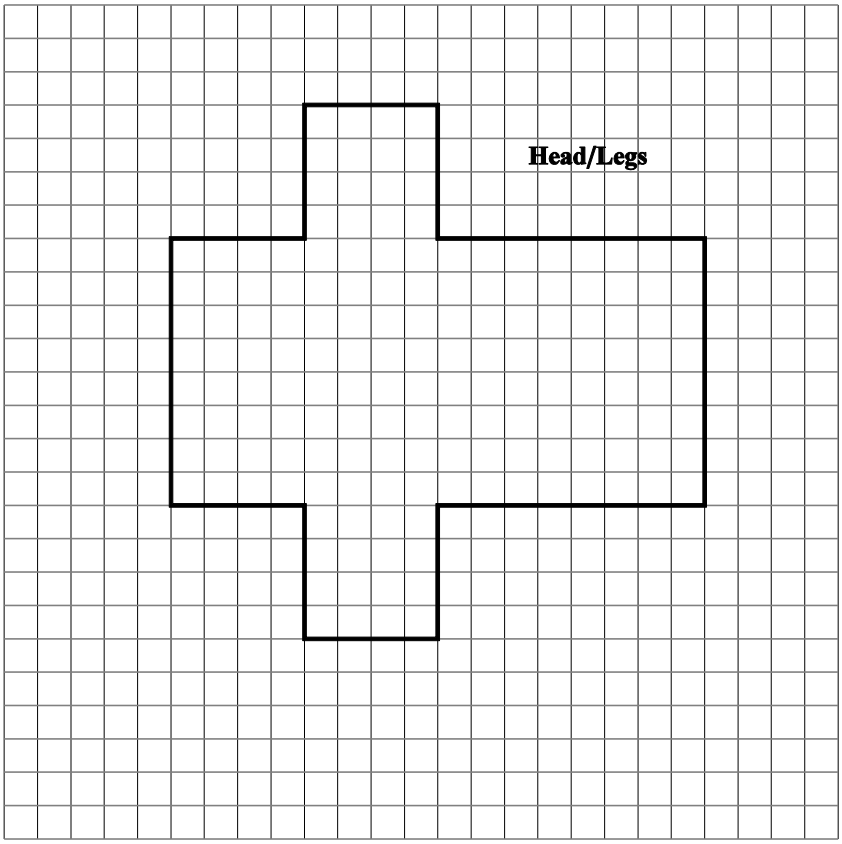


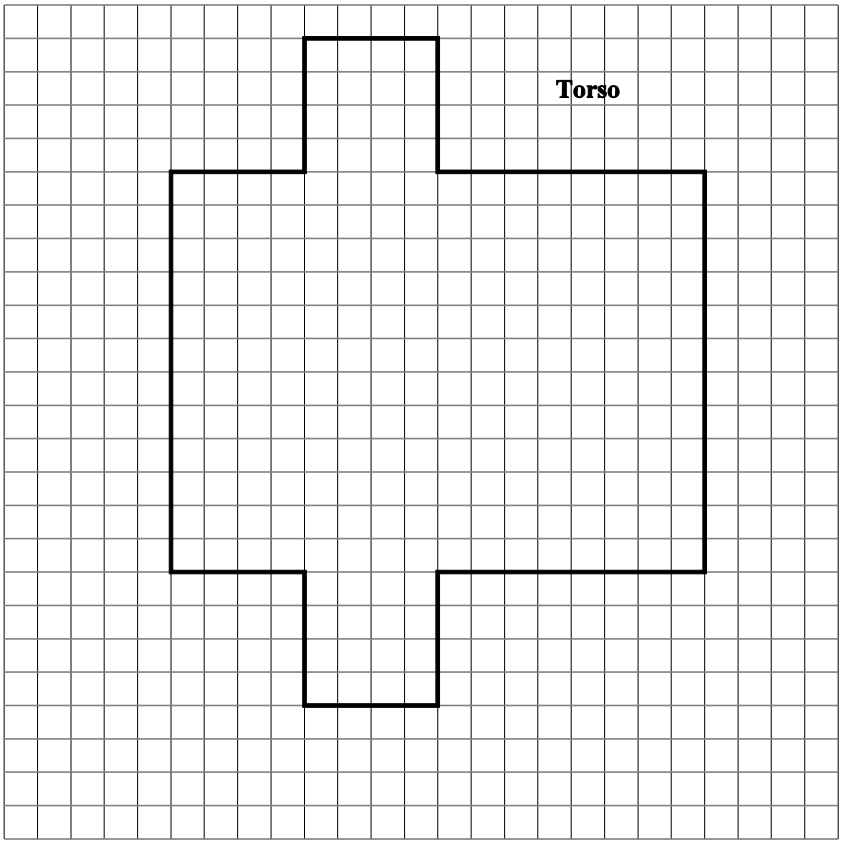
8. Sample net for scaling each net by a factor of 3.





9. Sample net for scaling each net by a factor of 4.





10.

Dimensions scaled by a factor of . . .	Surface area of the dog	Area scaled by a factor of . . .	Volume of dog	Volume scaled by a factor of . . .
1 (the unit dog)	54	1	13	1
2	216	4	104	8
3	486	9	351	27
4	864	16	832	64
5	1350	25	1625	125
n	$54n^2$	n^2	$13n^3$	n^3

11. For a scale factor of 50, the surface area is 135,000 square units and the volume is 1,625,000 cubic units. For a scale factor of 100, the surface area is 540,000 square units and the volume is 13,000,000 cubic units.

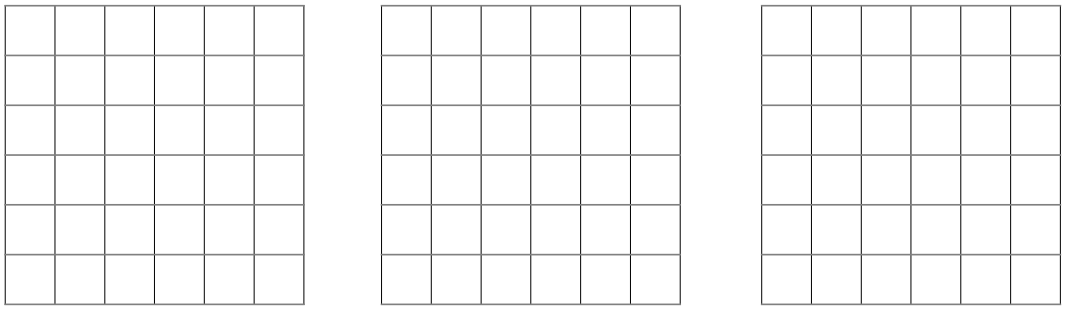
Unit Dog

1. As you built the original unit dog, how many linking cubes did it take to construct the entire dog?

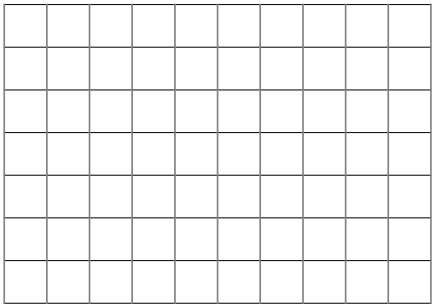
2. What is the volume of the original unit dog? Explain how to calculate the volume.

3. What is the surface area of the original unit dog? Explain how to calculate the surface area.

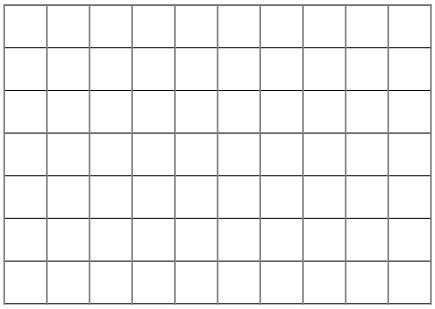
4. Draw the front, top, and side views of the original unit dog. Label each appropriately.



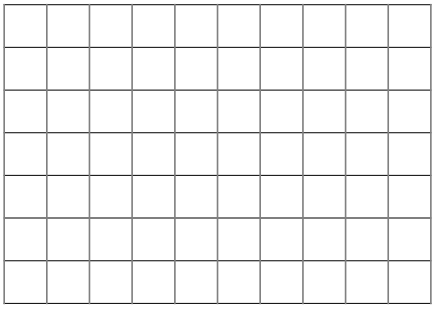
5. a. The head and legs of the original unit dog can be constructed from the same net. Sketch a net of the original unit dog's head or leg on the grid.

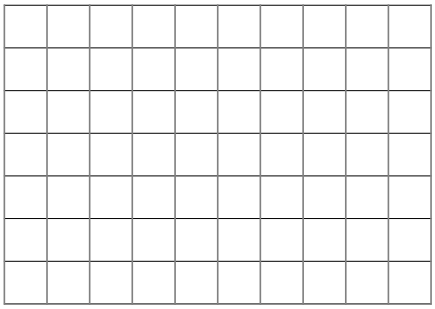


b. The original unit dog's torso can be constructed from a different net. Sketch a net of the original unit dog's torso on the grid.

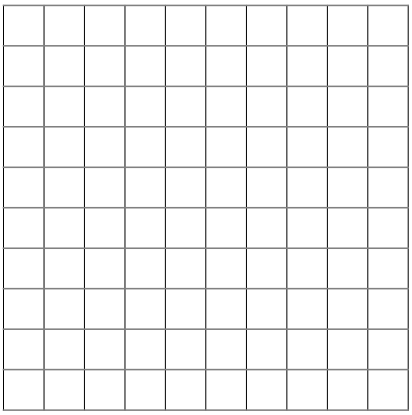


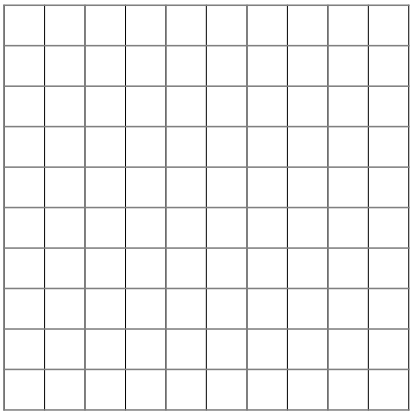
6. Compare your nets to those drawn by other class members. As a class, decide on a net that will be used for the head/leg and the torso and sketch these nets on the grids provided.



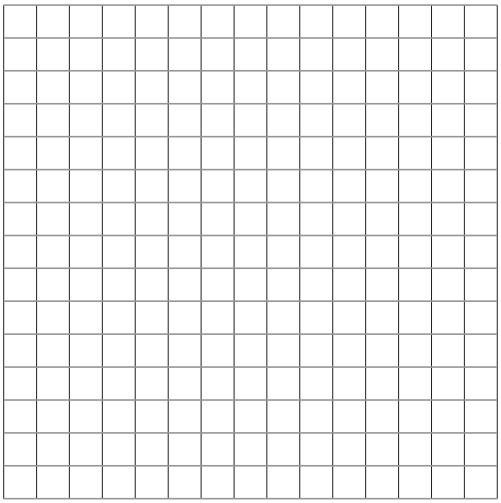


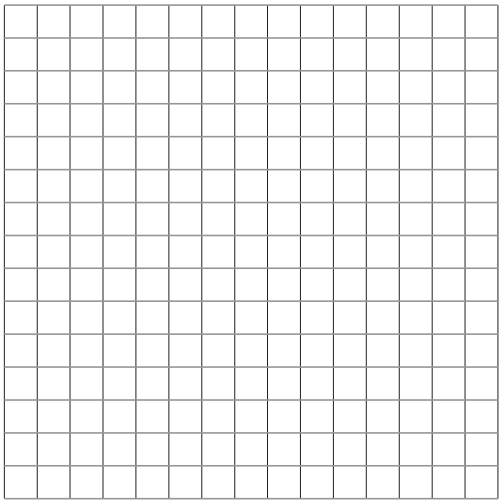
7. Using a scale factor of two, sketch a net of the scaled dog's head/leg and torso on the grid.



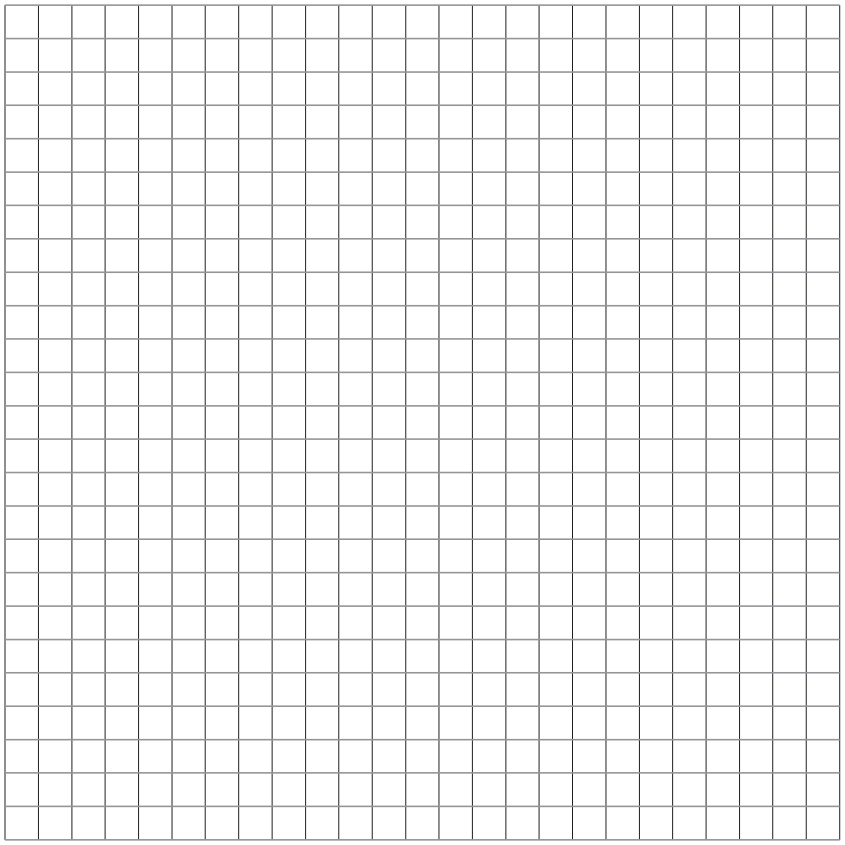


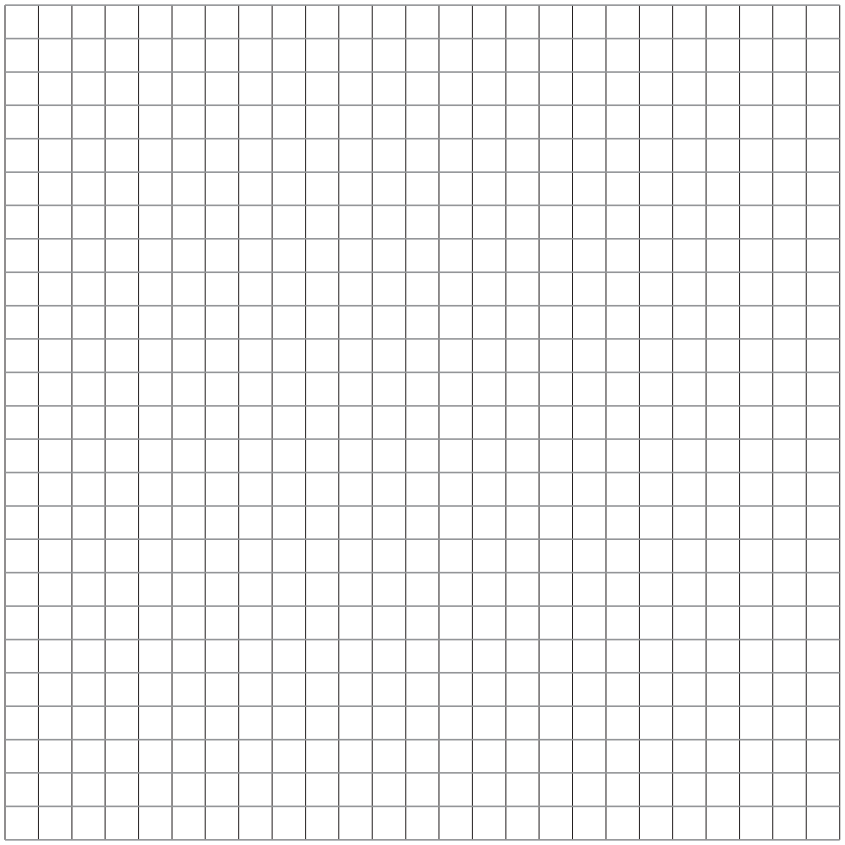
8. Using a scale factor of three, sketch a net of the scaled dog's head/leg and torso on the grid.





9. Using a scale factor of four, sketch a net of the scaled dog's head/leg and torso on the grid.





10. Complete the row for your assigned unit dog. Add the data from the other unit dogs created by your class.

Dimensions scaled by a factor of . . .	**Surface area of the dog**	**Area scaled by a factor of . . .**	**Volume of dog**	**Volume scaled by a factor of . . .**
1 (the unit dog)		1		1
2				
3				
4				
5				
n				

11. Use the information from the table to determine the surface area and volume for a dog when the dimensions are scaled with a factor of 50. What is the surface area and volume for a dog with a scale factor of 100?

Cardstock Copy : Unit Dog 1-inch Grid (7 x 10)