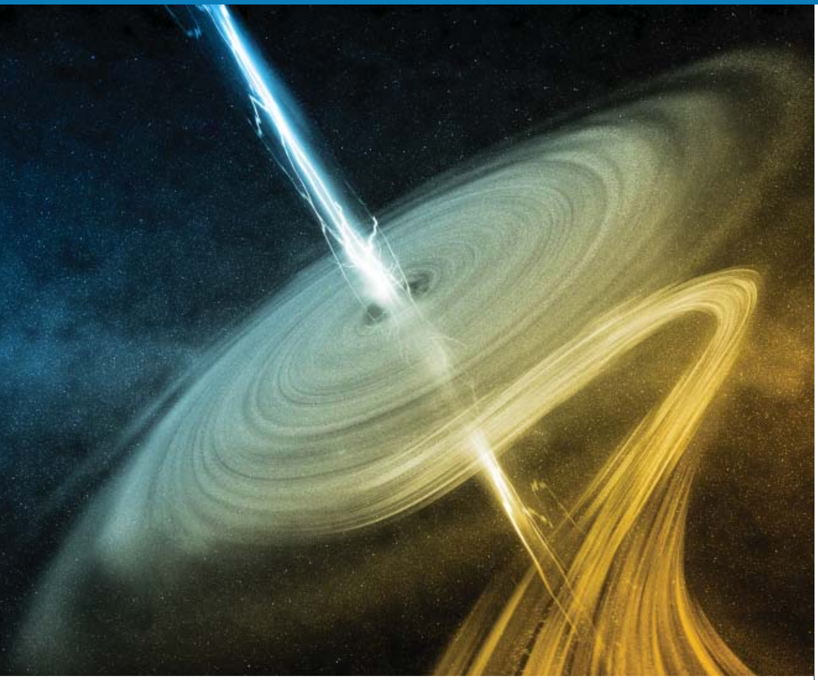


Beyond the Black Hole

Modeling an Imploding Star

MATERIALS AND RESOURCES

EACH GROUP

balance
calculator
meter stick
push pin
string
aluminum foil
balloon

ABOUT THIS LESSON

This lesson is designed to familiarize students with the concepts of volume, mass and density while relating these concepts to imploding stars and black holes.

OBJECTIVES

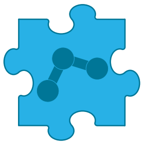
Students will:

- Model an imploding star to investigate the properties and formation of a black hole
- Calculate the radius of the "star," the volume of the "star," and the density of the "star"

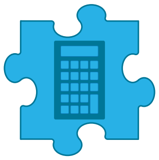
LEVEL

Middle Grades: Earth

NEXT GENERATION SCIENCE STANDARDS



DEVELOPING AND USING MODELS



USING MATHEMATICS

SYSTEMS AND SYSTEM MODELS

ESS1: THE UNIVERSE

ASSESSMENTS

The following types of formative assessments are embedded in this lesson:

- Visual assessment of student measuring techniques

The following assessments are located on our website:

- Middle Grades Earth: Space Science—Stars and Universe Assessment

COMMON CORE STATE STANDARDS

(LITERACY) RST.6-8.1

Cite specific textual evidence to support analysis of science and technical texts.

(LITERACY) RST.6-8.3

Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

(LITERACY) WHST.6-8.1

Write arguments focused on discipline-specific content.

(MATH) 6.EE.C

Represent and analyze quantitative relationships between dependent and independent variables.

(MATH) 7.EE.B

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

(MATH) 7.NS.A

Apply and extend previous understandings of operations with fractions.

(MATH) 7.RP.A

Analyze proportional relationships and use them to solve real-world and mathematical problems.

(MATH) 8.EE.A

Work with radicals and integer exponents.

(MATH) 8.EE.B

Understand the connections between proportional relationships, lines, and linear equations.

(MATH) 8.EE.C

Analyze and solve linear equations and pairs of simultaneous linear equations.

(MATH) 8.F.A

Define, evaluate, and compare functions.

(MATH) 8.F.B

Use functions to model relationships between quantities.

(MATH) 8.G.C

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres

TEACHING SUGGESTIONS

This lesson is designed to familiarize students with the concepts of volume, mass, and density and then relate these concepts to imploding stars on their way to becoming black holes. Prior to this activity, introduce the stages of the life cycle of a star to your students, either through lecture, reading assignment, or online search.

In addition, there are several words used to discuss the ideas surrounding black holes that your students may not be familiar with. *Space-time*, for instance, can be thought of as a continuum of four dimensions (three space and one of time) in which any object or event can be located. In a way, space-time is an area so large that you have to include the concept of time to understand it. For example, when you see the star Alpha Centauri in the night sky, you are seeing it as it was four years ago because it has taken four years for the light to reach Earth. Therefore, when you look into space you are looking back in time, which means time and space (space-time) are inseparable.

It is easiest to imagine the curvature of space-time by thinking of a flat rubber sheet that is stretched by the presence of massive objects. For example, a flat rubber sheet represents empty space-time. However, if a bowling ball were placed in the center of the rubber sheet, the sheet would now be warped. A marble traveling close by the warped rubber sheet would naturally follow a curved path down toward the bowling ball.

The bowling ball and the marble represent matter, and the rubber sheet represents space-time. Just as the bowling ball warped the rubber sheet causing the marble to fall toward it, so too a massive object like the sun warps space-time causing smaller massive objects like Earth to fall toward it. In summary, space-time tells matter how to move, and matter tells space-time how to curve.

Be sure to read the introduction to the lab with your students and help them understand the text. Share with students that they are going to model an imploding star on its way to becoming a black hole by using aluminum foil and balloons. They will determine the circumference, radius, volume, mass, and density of their "star" (aluminum foil balloon).

Blow up a balloon until the circumference is roughly 45 cm and tie the end off. Tell the students that this is the core of the "star." Cover the inflated balloon with aluminum foil and tell the students that the foil represents the outer layers of their "star." Demonstrate with a string how to measure the circumference of the balloon. Remind students to make three measurements and average their results. In addition, they should determine the mass of the "star" and record their initial measurements of mass and circumference in the data table.

The students are now ready to simulate the star collapsing inward toward its core. Squeeze the balloon with your hands but do not pop it. Tell the students to imagine that the force you apply to the foil covered balloon with your hands is the force of the enormous mass of the star collapsing inward on the core. Point out that when the forces are balanced, gravity is pulling everything inward but the nuclear reaction of fusion is pushing everything outward. However, when the star runs out of fuel then gravity wins the battle. There is no longer a force opposing gravity and the star collapses. If the star is massive, then it may continue to collapse and become a black hole.

TEACHING SUGGESTIONS (CONTINUED)

Take a push-pin and pop the balloon. You might want to tell the students to say “Supernova!” and then pop the balloon. Caution the students that they will need to gently shape the aluminum foil back into a “sphere” once they have popped their balloon, so they should not be too hasty in squashing their “imploding star.” Watch the students to be sure that they do not crush the foil entirely after the star is initially popped. It is important that the balloons have four different sizes and that the students make the foil ball smaller and smaller each time. In fact, you might suggest to them by Trial 4 that they roll the ball on the table to make it smaller.

Walk around the room and make sure the students realize that the mass is not changing as they squeeze the sphere smaller and smaller but the density is increasing. The change in density should be greater by a factor of 10 to 100. Students may observe a very small change in mass, especially if they lose part of the balloon. Because the amount of foil is not changed by its crushing, any mass change should be minimal at best. If there is a difference in mass, it is probably due to experimental error. Depending on your students’ math ability, they may need some assistance with the formulas.

A common misconception is that a black hole is made of matter that has been compacted to an incredibly small size. This is the major weakness of our model. The reality is that a star’s matter is destroyed at the center of a black hole, a point we call the singularity. Space-time is infinitely warped at this point of infinite density and zero volume. At the singularity of a black hole, there is nothing left except warped space-time. Time slows down as you approach the edge of the black hole, the so-called *event horizon*. Inside the event horizon, time flows toward and into the singularity dragging everything that’s inside the horizon forward in time to its ultimate destruction.

Looking at a black hole from the outside, it will bend light rays that pass near it. A black hole is not a cosmic vacuum cleaner, however. If our Sun suddenly became a black hole of the same mass, our orbit around the Sun would remain unchanged. To be pulled into a black hole, an object must cross inside the *Schwarzschild radius;* once inside that radius, even light cannot escape. The Schwarzschild radius depends upon the mass of the object. For our Sun, the Schwarzschild radius is 3 km or 1.86 miles. To quote Dante, “Abandon all hope, ye who enter here,” as all information appears to be lost if you venture within the event horizon of a black hole.

In the Going Further section, students determine the radius and density of their “star” if it were to collapse into a black hole. Use the Schwarzschild radius,

$$R = \frac{2GM}{c^2} \qquad \text{(Eq. A)}$$

where

R is the radius of the event horizon
G is the universal gravitational constant, $6.67 \times 10^{-8}\ \text{cm}^3/\text{g}\cdot\text{s}^2$
M is the mass of the object
c is the speed of light, 3×10^{10} cm/s

This equation basically says that any object can be a black hole if it gets small enough. In nature, this only happens with massive stars.

TEACHING SUGGESTIONS (CONTINUED)

Using the mass of the sample data “imploding star,”

$$R = \frac{2GM}{c^2} = \frac{2\left(6.67\times10^{-8}\ \frac{\text{cm}^3}{\text{g}\cdot\text{s}^2}\right)(11.3\ \text{g})}{\left(3\times10^{10}\ \frac{\text{cm}}{\text{s}}\right)^2} = 1.67\times10^{-27}\ \text{cm}$$

$$V = \frac{4}{3}\pi r^2 = \frac{4}{3}\pi(1.67\times10^{-27}\ \text{cm}) = 1.968\times10^{-80}\ \text{cm}^3$$

$$\text{density} = \frac{11.3\ \text{g}}{1.968\times10^{-80}\ \text{cm}^3} = 5.74\times10^{80}\ \text{g/cm}^3$$

The radius of the aluminum foil would have to be 1.67×10^{-27} cm (remind the students the average diameter of an atom is 1.75×10^{-13} cm) and the density would have to be 5.74×10^{80} g/cm^3 which is impossible to imagine. The point is that a black hole is not matter that has been compacted into a smaller and smaller space, but a warping of space-time.

The most dense object that students will probably be able to come up with is gold at around 19.3 g/cm^3. You might share with them that the average density of Earth is 5.5 g/cm^3, the average density of our Sun is 150 g/cm^3, and the density of a neutron or proton is about 1.5×10^{15} g/cm^3. Because an atom is mostly empty space, there is no comparison between our model of a collapsing star as it becomes a black hole and an actual black hole.

DATA AND OBSERVATIONS

Table 1. Black Hole Measurements

Trial	Circumference (cm)	Radius (cm)	Volume (cm³)	Mass (g)	Density (g/cm³)
1	45.0	7.16	1540	12.0	0.00779
2	40.1	6.38	1090	11.4	0.0104
3	35.1	5.59	732	11.4	0.0156
4	20.0	3.18	135	11.3	0.0837
5	4.49×10^{-12}	7.15×10^{-13}	1.53×10^{-36}	11.3	7.38×10^{36}

ANALYSIS AND CONCLUSION QUESTIONS

1. What stage of a star’s life cycle is represented when you popped the “star” balloon?

   The popping of the balloon “star” represents a supernova.

2. During its life, a star is held together by the inward, pulling force of gravity and the outward, pushing force of nuclear fusion. What force wins this battle, when you popped the “star” balloon?

   The gravitational force wins the battle.

3. What kind of stars become black holes: small mass, medium mass, or large mass? Explain your answer.

   Large-mass stars become black holes because it takes a huge mass to collapse to the form the density of a black hole

4. Describe any observed changes in the circumference of your “imploding star” as you completed each trial.

   After each trial, the “imploding star” decreased in size or circumference.

5. Compare the mass and density of your “star” to the mass and density of your “imploding star.”

   The mass of the star and the imploding star are relatively the same whereas the density of the “imploding star” is much greater than the star, 10 times greater.

6. Compare the mass and density of your “imploding star” when you had crushed it as much as possible to its mass and density if you were able to crush it to the same size as the atomic diameter of an aluminum atom.

   The mass of both remains relatively the same whereas the difference in density is an incredible 10^{38} times greater.

   There is no comparison between the density of our imploding star and that of a black hole. In fact, the matter in a black hole is not matter that we are familiar with that has been compacted into a small space, but space-time that is infinitely warped as matter is destroyed at the singularity.

ANALYSIS AND CONCLUSION QUESTIONS (CONTINUED)

7. Models and simulations are useful because they allow us to study phenomena that are otherwise too small, too far away, or too complicated to observe. Few models can replicate the actual phenomenon perfectly, however. Identify and explain one limitation of "Beyond the Black Hole."

   The major limitation of the model is that a black hole is not made of matter that has been compacted to an incredibly small size. At the singularity all matter is destroyed, so that space-time is infinitely warped.

GOING FURTHER

Use the Schwarzschild radius,

$$R = \frac{2GM}{c^2} \qquad \text{(Eq. 4)}$$

where

R is the radius of the event horizon
G is the universal gravitational constant, 6.67×10^{-8} cm^3/g·s^2
M is the mass of the object
c is the speed of light, 3×10^{10} cm/s

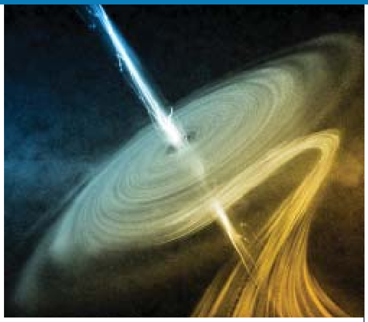
Equation 4 basically says that any object can be a black hole if it gets small enough. However, in nature this only happens with massive stars.

Using the mass of your "imploding star," calculate its radius and density if it were to become a black hole.

$$R = \frac{2GM}{c^2} = \frac{2\left(6.67\times10^{-8}\ \frac{\text{cm}^3}{\text{g}\cdot\text{s}^2}\right)(11.3\ \text{g})}{\left(3\times10^{10}\ \frac{\text{cm}}{\text{s}}\right)^2} = 1.67\times10^{-27}\ \text{cm}$$

$$V = \frac{4}{3}\pi r^2 = \frac{4}{3}\pi(1.67\times10^{-27}\ \text{cm}) = 1.968\times10^{-80}\ \text{cm}^3$$

$$\text{density} = \frac{11.3\ \text{g}}{1.968\times10^{-80}\ \text{cm}^3} = 5.74\times10^{80}\ \text{g/cm}^3$$



Beyond the Black Hole

Modeling an Imploding Star

NATIONAL
MATH + SCIENCE
INITIATIVE

MATERIALS

- *balance*
- *calculator*
- *meter stick*
- *push pin*
- *string*
- *aluminum foil*
- *balloon*

What is a black hole? A **black hole** is a region of warped space and time from which not even light can escape. But what does that mean? Einstein's general theory of relativity describes gravity as geometry, a curvature of space-time (caused by the presence of matter). If the curvature of space-time is fairly weak, then Newton's idea of gravitational force explains the regular motion of the planets with great success. However, incredibly massive objects warp the fabric of space-time dramatically.

A black hole may be formed from an imploding star. During its life, a star is held together by the inward, pulling force of gravity and the outward, pushing force of nuclear fusion. For a very massive star at the end of its life, when light elements such as helium are no longer produced by fusion, the star runs out of fuel and collapses as gravity wins the battle.

If the star is just the right size, then it is crushed into a smaller and smaller volume so that the gravitational force of attraction increases. The escape velocity or the minimum speed needed to break free from the intense gravitational field increases as well. Eventually a point is reached when even light, which travels at 3×10^8 meters per second, is not traveling fast enough to escape. Consequently, if light cannot escape we call it a black hole.

A common misconception is that a black hole is made of matter that has been compacted to an incredibly small size. The reality is that a star's matter is destroyed at the center of a black hole, a point we call the **singularity**. Space-time is infinitely warped at this point of infinite density and zero volume. At the singularity of a black hole there are no electrons, protons, or neutrons or any type of matter that we are familiar with, nothing except warped space-time.

Time slows down as you approach the edge of a black hole, the so-called **event horizon**. Inside the event horizon, time flows forward. Everything that's inside the event horizon is pulled toward the singularity and its ultimate destruction. Looking at a black hole from the outside, it will bend light rays that pass near it.

However, a black hole is not a cosmic vacuum cleaner. If our Sun suddenly became a black hole of the same mass, our orbit around the Sun would remain unchanged. To be pulled into a black hole, you must cross inside the **Schwarzschild radius,** and once inside that radius, well, nice knowing you.

The Schwarzschild radius depends upon the mass of the object. For our Sun, the Schwarzschild radius is 3 km or 1.86 miles. To quote Dante, "Abandon all hope, ye who enter here," as all information appears to be lost if you venture within the event horizon of a black hole.

Do black holes actually exist? Most physicists believe they do, basing their views on a growing body of observations, primarily x-ray radiation falling into the black hole. In fact, present theories of how the cosmos began rest in part on Einstein's work that predicts the existence of black holes. Yet Einstein himself denied their existence, believing that black holes were a mere mathematical curiosity. He died in 1955, before the term "black hole" was coined or understood and observational evidence for black holes began to surface.

PURPOSE

To model a collapsing star on its way to becoming a black hole.

SAFETY ALERT!

- Be careful when using the push-pin to pop the balloon—the push-pin is sharp.

PROCEDURE

1. Inflate the balloon until the circumference is roughly 45 cm.

   To measure the circumference, wrap a string around the middle of the balloon. Use your finger to mark the length of the string. Measure the length of the string using a meter stick.

   When you have inflated your balloon to the proper circumference, tie off the end of the balloon.
2. Carefully cover the inflated balloon with aluminum foil. Make sure to cover the entire surface of the balloon.

Figure 1. Determining the mass of your "star"

3. Using the string, measure the circumference of the "star" three times at three different locations. Take the average of the circumferences and record the average in Table 1 under Trial 1.
4. Place the "star" on the triple beam balance and determine the mass (Figure 1). Record the mass in Table 1 under Trial 1.
5. Using a push-pin, carefully pop the "star." This represents the stage of a star becoming a supernova.
6. Now that the "star" is popped, carefully crush the foil so that it is *slightly* smaller than its original volume.
7. Measure and record the circumference of the "star" three times at three different locations. Take the average and record this circumference in Table 1 under Trial 2.

   Be careful not to crush the "star" further until directed to do so.

IMPORTANT! *Do not crush the foil while popping the "star." You will crush the foil in stages.*

Remember that the diameter is twice the radius.

PROCEDURE (CONTINUED)

8. Measure the mass of the "star" on a balance (Figure 2). Record the mass under Trial 2 in Table 1.

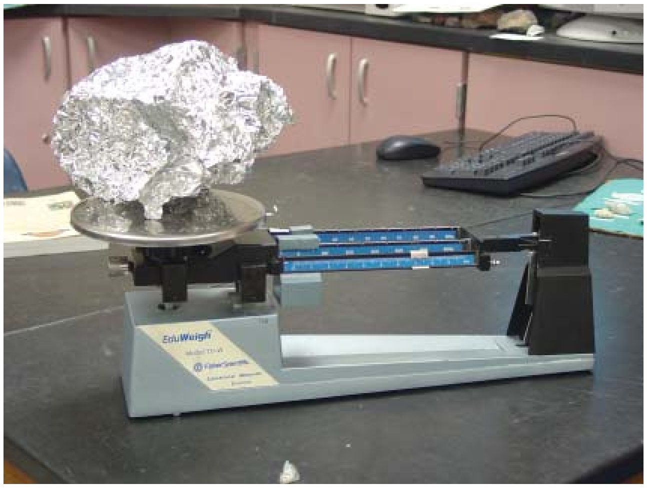


Figure 2. Determining the mass of your "star"

9. Crush the "star" slightly once again so that it is not quite completely crushed.

   Measure the new circumference of the "star" three times at three different locations. Take the average and record this circumference under Trial 3 in Table 1.

10. Crush the "star" once again so that it is now as compact as possible (Figure 3).

    Measure and record the new average circumference and mass under Trial 4 in Table 1.

Figure 3. Determining the mass of your "star"

11. Assume that nature allowed you to compact the aluminum foil into a sphere the same size as the atomic diameter of an aluminum atom (diameter of 1.43×10^{-12} cm). Record this circumference, radius, volume, mass, and density under Trial 5 in Table 1.

DATA AND OBSERVATIONS

Table 1. Black Hole Measurements

Trial	Circumference (cm)	Radius (cm)	Volume (cm^3)	Mass (g)	Density (g/cm^3)
1					
2					
3					
4					
5					

ANALYSIS

Use the values from Table 1 and the formulas of Equations 1–3 to calculate the radius of the "star," the volume of the "star," and the density of the "star."

Record these values in Table 1.

$$\text{circumference} = 2\pi r \qquad \text{(Eq. 1)}$$

$$\text{volume} = \frac{4}{3}\pi r^3 \qquad \text{(Eq. 2)}$$

$$\text{density} = \frac{m}{v} \qquad \text{(Eq. 3)}$$

$$\pi = 3.14$$

ANALYSIS AND CONCLUSION QUESTIONS

1. What stage of a star's life cycle is represented when you popped the "star" balloon?

2. During its life, a star is held together by the inward, pulling force of gravity and the outward, pushing force of nuclear fusion. What force wins this battle, when you popped the "star" balloon?

3. What kind of stars become black holes: small mass, medium mass, or large mass? Explain your answer.

4. Describe any observed changes in the circumference of your "imploding star" as you completed each trial.

5. Compare the mass and density of your "star" to the mass and density of your "imploding star."

ANALYSIS AND CONCLUSION QUESTIONS (CONTINUED)

6. Compare the mass and density of your “imploding star” when you had crushed it as much as possible to its mass and density if you were able to crush it to the same size as the atomic diameter of an aluminum atom.

7. Models and simulations are useful because they allow us to study phenomena that are otherwise too small, too far away, or too complicated to observe. Few models can replicate the actual phenomenon perfectly, however. Identify and explain one limitation of “Beyond the Black Hole.”

GOING FURTHER

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